積分練習問題

(1) $\int_0^1 (x^6 + x^4)dx$

$$=\left[\frac{1}{7}x^7 + \frac{1}{5}x^5\right]_0^1$$

$$=\left(\frac{1}{7} + \frac{1}{5}\right) - \left(\frac{1}{7} \cdot 0 - \frac{1}{5} \cdot 0\right)$$

$$=\frac{12}{35}$$

(2) $\int_0^{\frac{\pi}{6}} \sin\left(2x + \frac{\pi}{6}\right)dx$

$$=\left[-\frac{1}{2}\cos\left(2x + \frac{\pi}{6}\right)\right]_0^{\frac{\pi}{6}}$$

$$=\left(-\frac{1}{2}\cos\frac{3}{6}\pi\right) - \left(-\frac{1}{2}\cos\frac{\pi}{6}\right)$$

$$=\frac{1}{2}$$

(3) $\int_4^6 \frac{dx}{x^2-2x-3}$

$$=\int_4^6 \frac{dx}{(x-3)(x+1)}$$

$$=\int_4^6 \frac{1}{2}\left(\frac{1}{x-3} - \frac{1}{x-1}\right)dx$$

$$=\frac{1}{2}\left[\log(x-3) - \log(x-1)\right]_4^6$$

$$=\log 3 - \frac{1}{2}\log 5$$

(4) $\int_{e}^{e^2} \log x \, dx$

$= [x \log x - x]_{e}^{e^2}$

$= (e^2 \log e^2 - e^2) - (\log e - e)$

$= e^2 + e - 1$

(5) $\int_{\square}^{\square} \sin^2 \theta \, d\theta$

$= \int \frac{1 - \cos 2\theta}{2} d\theta$

$= \frac{1}{2} \theta - \frac{1}{4} \sin 2\theta + C$ (Cは積分定数)